Delta Modulation

Status Report

January 1, 1972 - July 1, 1972

Prepared for

National Aeronautics and Space Administration

Manned Spaceflight Center

Houston, Texas

under

NASA Grant NGR 33-013-063

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Introduction

This status report summarizes several of the areas of research in the area of Delta Modulation being supported under NASA Grant NGR - 33-013-063 during the period January 1, 1972 - July 1, 1972.

A paper describing the basic delta modulation systems studied has been published in the December 1971 issue of the IEEE Transactions on Communication Technology. A second paper, describing the systems' response to voice and video was presented at the International Communications Conference in June 1972, and a third paper describing several new delta modulation systems used for voice and video is scheduled for presentation at the National Telecommunications Conference in December 1972.

I. Stability of the Song Delta Modulator

The Song Delta Modulator shown in Fig 1 has been operated in two modes. These modes can best be described using Fig 2. Here we see that the output of function generator g_1 is

$$g_1 = \begin{cases} (x_k - x_{k-1}) \text{ sgn } (e_k) & |x_k - x_{k-1}| \geq 2S \\ 2\,S \text{ sgn } (e_k) & |x_k - x_{k-1}| < 2S \end{cases} \tag{1}$$

where S is the minimum step-size.

The output of function generator g_2 can be altered depending on whether voice or video is to be encoded. If voice is to be encoded we select

$$g_2 = \begin{cases} S \text{ sgn } (e_{k-1}) & |x_k - x_{k-1}| \geq 2\,S \\ 0 & |x_k - x_{k-1}| < 2\,S \end{cases} \tag{2}$$

If video is to be encoded

$$g_2 = \begin{cases} \frac{1}{2} (x_k - x_{k-1}) \text{ sgn } (e_{k-1}) & |x_k - x_{k-1}| \geq 2\,S \\ 0 & |x_k - x_{k-1}| < 2\,S \end{cases} \tag{3}$$

It is noted that for $|x_k - x_{k-1}| \geq 2S$ the slope of g_1 is unity while the slope of g_2 is either zero or one-half. It is proven below that if the slope of g_1 is α and the slope of g_2 is β then instability results if

$$\alpha^2 - \beta^2 \geq 1 \tag{4}$$

Proof: Referring to Fig 1 we see that

$$g_1 (k) + g_2 (k) = x_{k+1} - x_k \tag{5}$$

i.e., the values of g_1 and g_2, when added, form the next value, $x_{k+1} - x_k$. For example, if $x_1 - x_o = 10S$ and $e_k = +1$, then $g_1 = +10S$ and $g_2 = +S$ (in the voice mode). Hence, $g_1 + g_2 = 11S$ and $x_2 - x_1 = 11S$. Since $x_k - x_{k-1}$ represents the instantaneous step-size, we see that in the

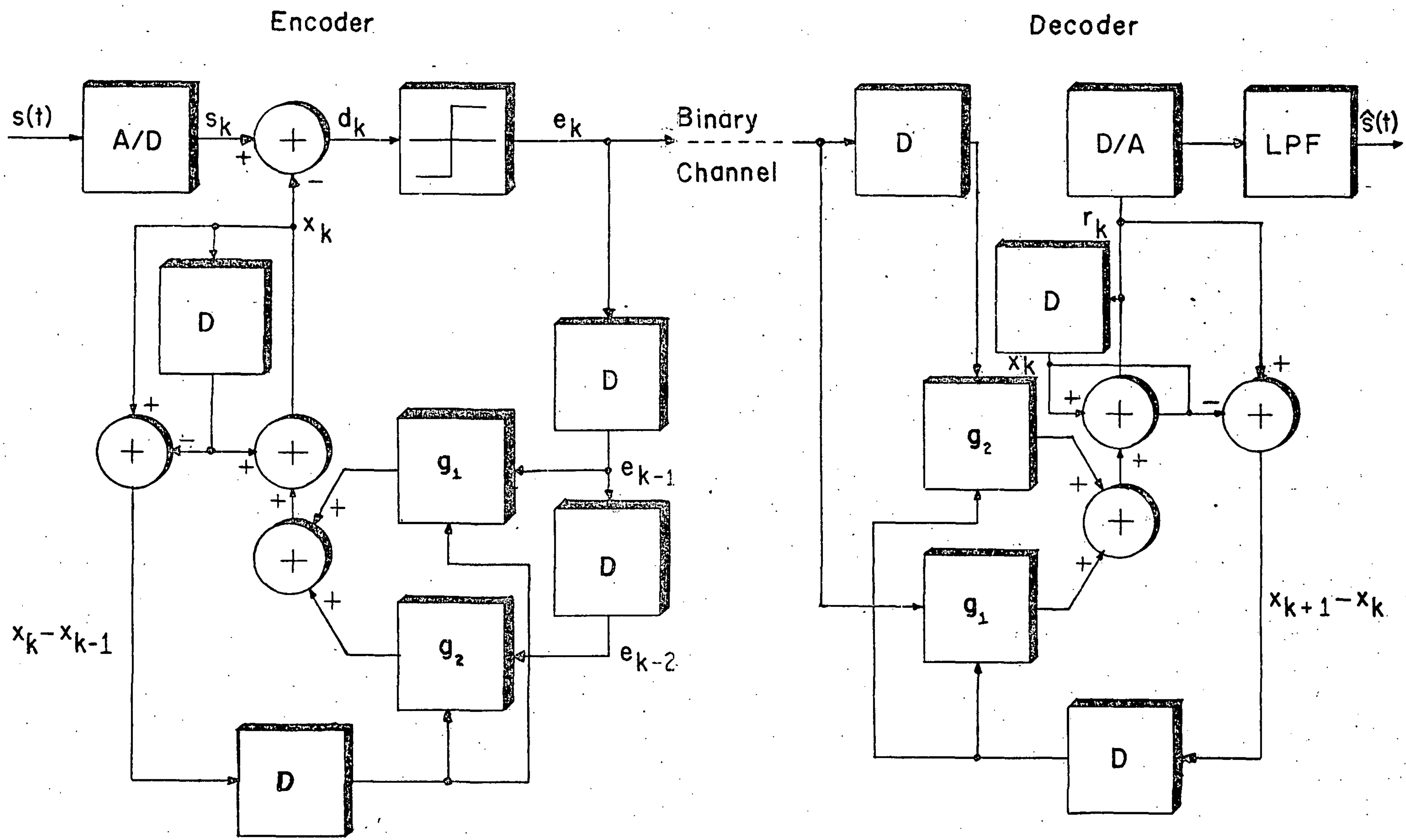
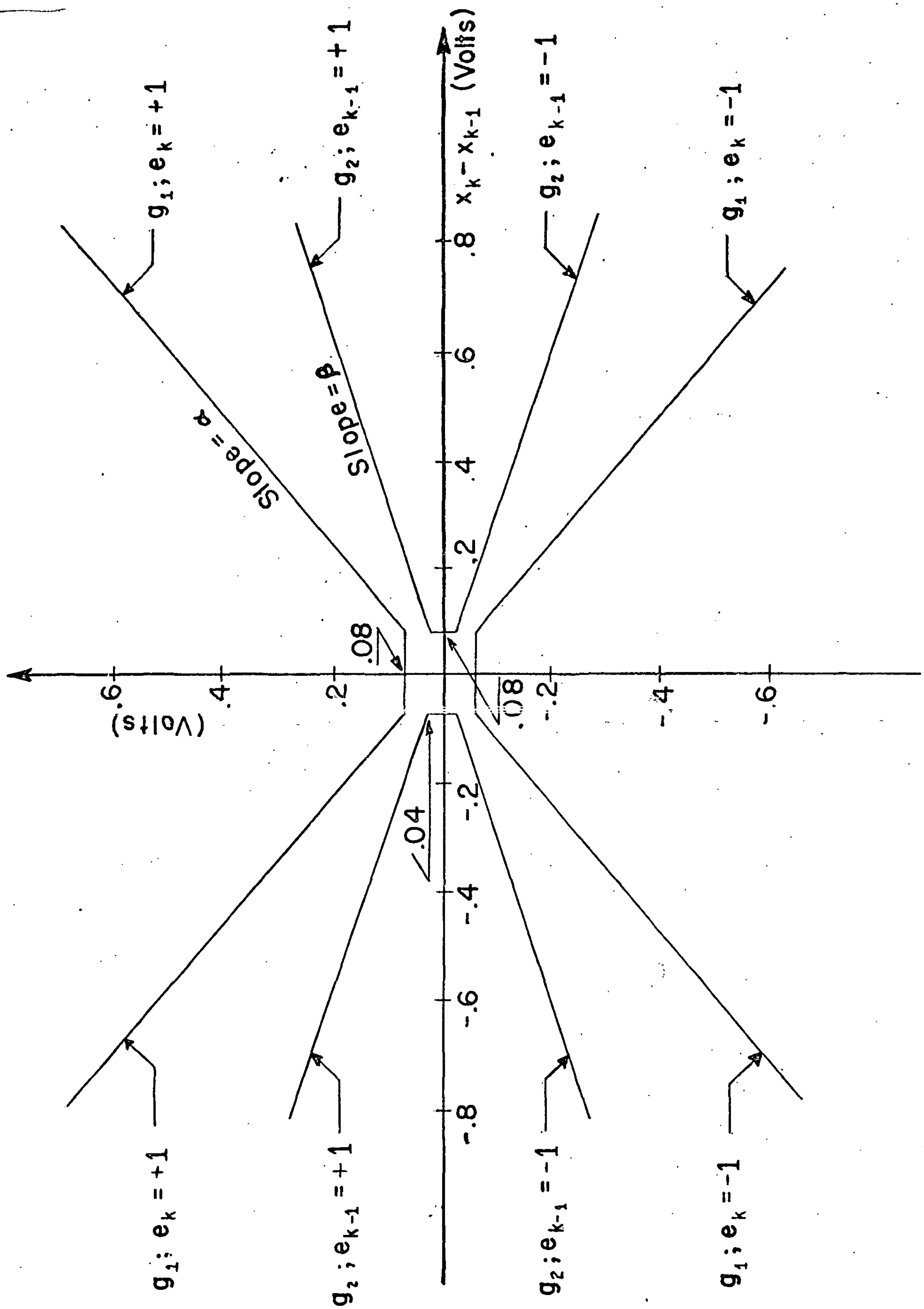

Encoder
Decoder
s(t)
A/D
s_k
d_k
e_k
Binary
Channel
D
D/A
LPF
$\hat{s}(t)$
x_k
r_k
e_{k-1}
e_{k-2}
g_1
g_2
$x_k - x_{k-1}$
$x_{k+1} - x_k$

Figure 1



$g_1 ; e_k = +1$
$g_2 ; e_{k-1} = +1$
$x_k - x_{k-1}$ (Volts)
$g_2 ; e_{k-1} = -1$
$g_1 ; e_k = -1$
Slope = α
Slope = β
(Volts)
.08
.08
.04
.8
.6
.4
.2
-.2
-.4
-.6
-.8

Figure 2

voice mode the step-size increases or decreases by S providing $|x_k - x_{k-1}| \geq 2S$.

Proof that $\alpha^2 - \beta^2 \geq 1$ Results in Instability

Assume that the slope of g_1 is α and the slope of g_2 is β. Now refer to Fig 3, Here we see that the estimates x_{k-3}, x_{k-2}, etc., are approaching m(t). If the system is stable, $x_{k+1} < x_{k-3}$. If the system is unstable $x_{k-3} - x_{k+1} \leq 0$. Since $e_{k-3} = e_{k-2} = -1$, we have from Fig 2,

$$|\Delta x_2| = (\alpha + \beta)\,|\Delta x_3| \tag{6}$$

Since $e_{k-1} = +1$,

$$|\Delta x_1| = (\alpha - \beta)\,|\Delta x_2| \tag{7}$$

and similarly since $e_k = +1$

$$|\Delta x_o| = (\alpha + \beta)\,|\Delta x_1| \tag{8}$$

Combining (6), (7) and (8) we have, for instability,

$$\begin{aligned} x_{k-3} - x_{k+1} &= |\Delta x_3| + |\Delta x_2| - |\Delta x_1| - |\Delta x_o| \\ &= \left[1 + (\alpha + \beta) - (\alpha - \beta)\,(\alpha + \beta - (\alpha + \beta)\,(\alpha + \beta)\,(\alpha + \beta) \right] |\Delta x_3| \\ &\leq 0 \end{aligned} \tag{9}$$

Thus

$$(1 + \alpha + \beta)\,(1 - \alpha^2 + \beta^2) \leq 0 \tag{10}$$

Since $\alpha, \beta \geq 0$ we have an unstable system if

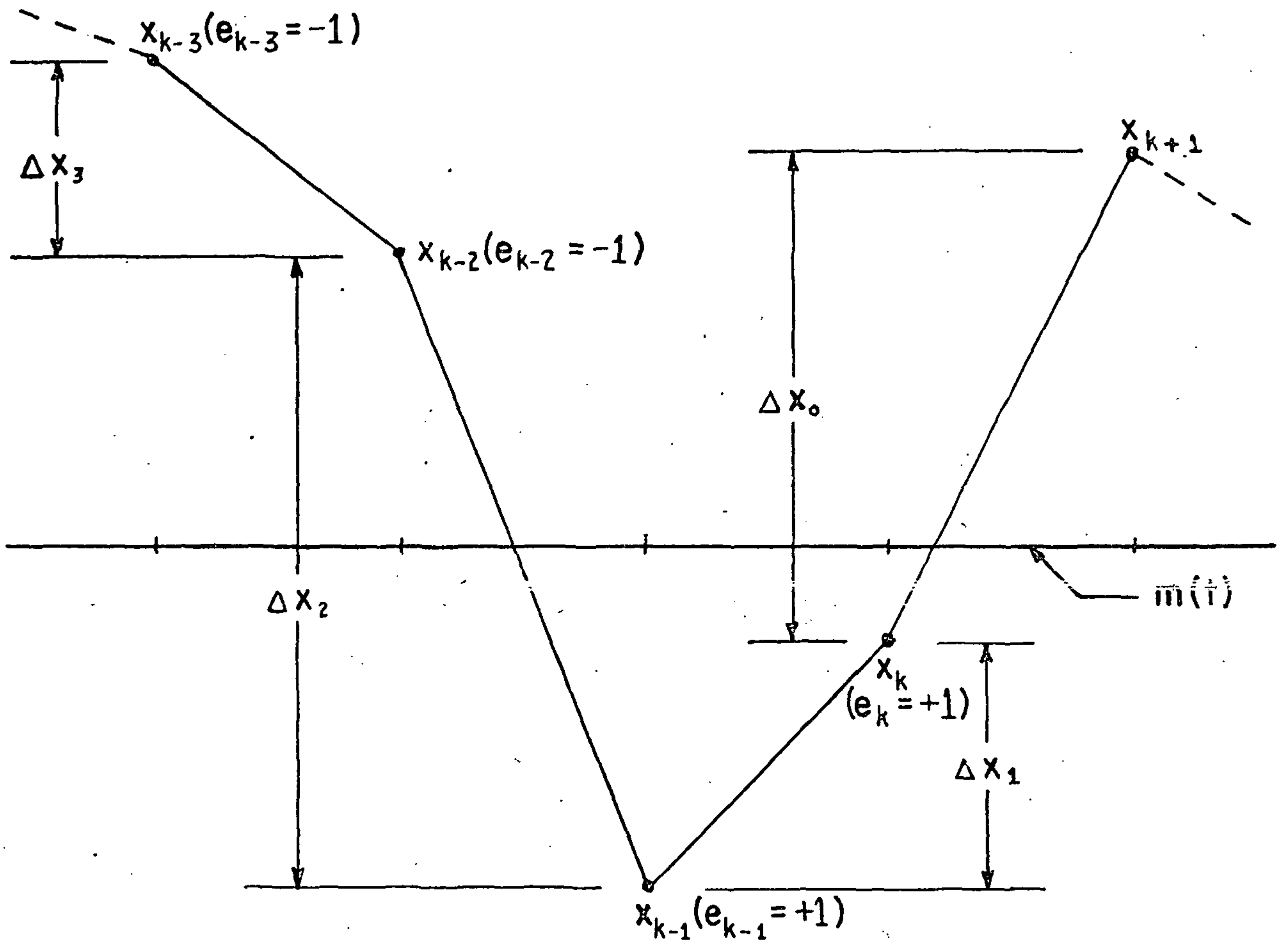
$$1 - \alpha^2 + \beta^2 \leq 0 \tag{11a}$$

or

$$\alpha^2 - \beta^2 \geq 1 \tag{11b}$$

Hence, for stability

$$\alpha^2 - \beta^2 < 1 \tag{12}$$



$x_{k-3}(e_{k-3}=-1)$
Δx_3
$x_{k-2}(e_{k-2}=-1)$
x_{k+1}
Δx_o
Δx_2
$\bar{m}(i)$
x_k
$(e_k=+1)$
Δx_1
$x_{k-1}(e_{k-1}=+1)$

Figure 3

Note that the procedure used for the Song mode: $\alpha = 1$ and $\beta = \frac{1}{2}$, is stable. The advantage of this system over other stable systems is that the step-size can increase by 50% ($\alpha + \beta = 1.5$) and decrease by 50% ($\alpha - \beta = 0.5$). This rapid rise and fall makes the Song mode of operation particularly suitable for video encoding.

Note also that the Enhanced Abate mode of operation, defined by $\alpha = 1$ and $\beta = 0$ is unstable since $\alpha^2 - \beta^2 = 1$. This instability is illustrated in Fig 4. Here we see that if $m(k-1) - x(k-1) \leq S$ then $x_k - x_{k-1} = \Delta x_1 + S$ and $x_{k+1} - x_k = -\Delta x_1$ so that $x_{k+2} - x_{k+1} = -\Delta x_1 - S$. As a result $x_{k+2} - x_{k-2} = 0$, and that system oscillates with a peak-to-peak amplitude $2 \Delta x_1 + S$, which can be quite large. Fortunately, the probability of $m(k-1) - x(k-1) \leq S$ is small and this situation does not occur in practice.

In the next section we discuss the Garodnick Modification which insures stability of the Abate mode.

II. Garodnick's Modification

In the previous section we described that operation of the Song DM when the instantaneous step-size $|x_k - x_{k-1}| \geq 2S$. In this section we consider steady state, or idle-channel, noise which occurs when $|x_k - x_{k-1}| \leq 2S$. We shall show that in this region the Song DM has a fundamental frequency of $f_S/4$, where f_s is the sampling frequency. Thus, if the sampling frequency of a voice signal is 9.6 Kilobits/sec a noise component exists at 2.4 KHz, and falls inband. It is conjectured that this effect is not significant when encoding video signals.

The Garodnick Modification places the fundamental frequency component of the idle-channel noise at $f_S/2$, which is the same as for the "linear" DM.

Steady-State Response of the Enhamed Abate and the Song DM

Figure 5a shows the steady state response of the Enhanced Abate mode of operation. Note that the steady state response is periodic with frequency $f_S/4$ after x_{k+1}, and that the peak-to-peak amplitude of the oscillation is 3S.

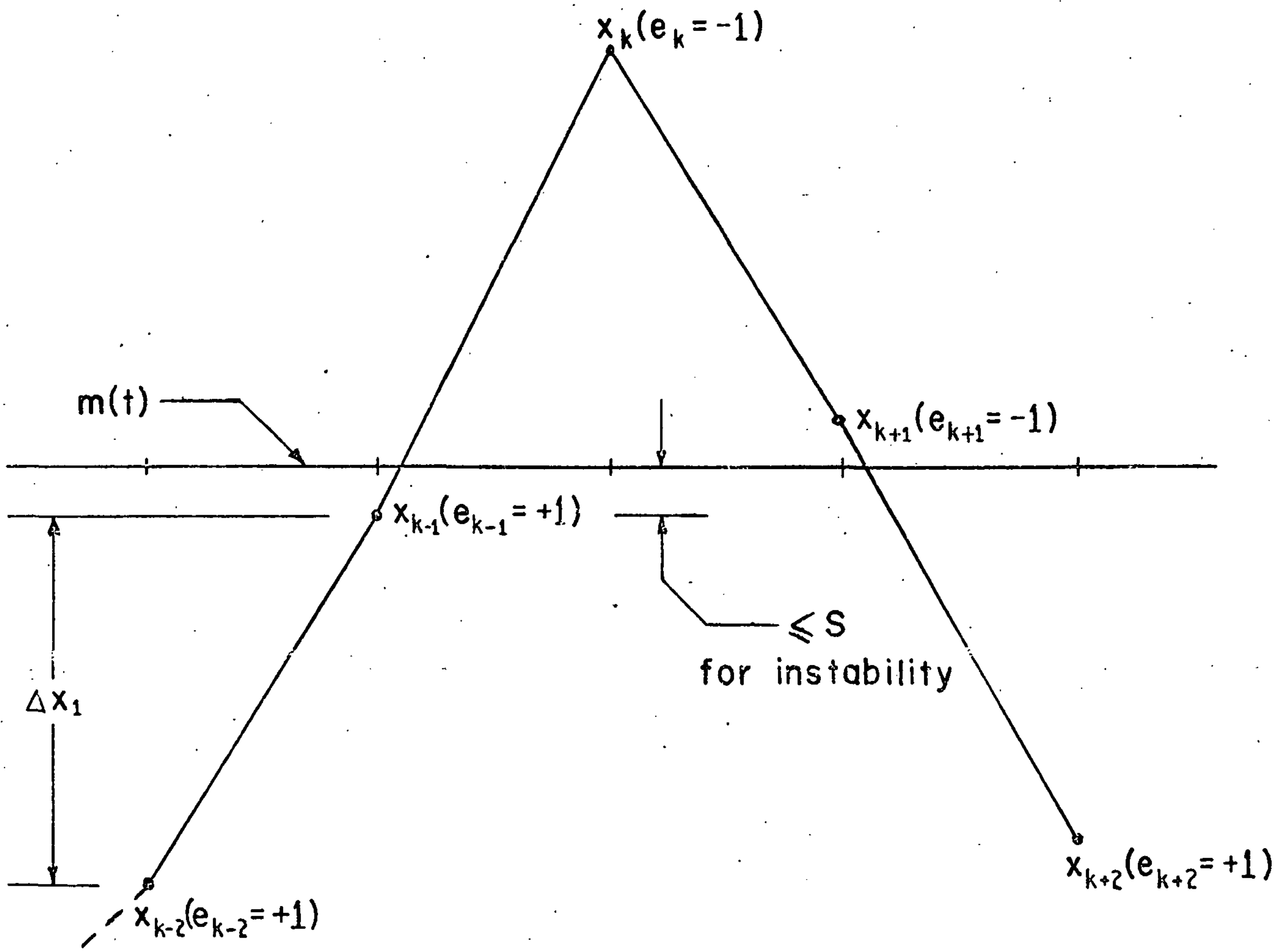

$x_k(e_k = -1)$
m(t)
$x_{k+1}(e_{k+1} = -1)$
$x_{k-1}(e_{k-1} = +1)$
$\leqslant S$
for instability
Δx_1
$x_{k+2}(e_{k+2} = +1)$
$x_{k-2}(e_{k-2} = +1)$

Figure 4

To see how this response is obtained assume that $x_k - x_{k-1} = 2S$ and $e_{k-1} = +1$ while $e_k = -1$ as shown. Then $g_1 = -2S$ and $g_2 = +1S$ (see Fig 2). Thus $x_{k+1} - x_k = (-2+1)S = -1S$. Now $e_{k+1} = +1$, hence $g_1 = +2$ and $g_2 = 0$. Thus $x_{k+2} - x_{k+1} = +2 + 0 = +2$; etc.

An alternate steady-state response is possible using the Abate mode. Consider, as shown in Fig 5b that $x_k - x_{k-1} = 3S$ and $e_{k-1} = +1$ while $e_k = -1$. Then $g_1 = -3S$ and $g_2 = +S$, so that $x_{k+1} - x_k = -2S$. If m(t) is such that $x_{k+1} > m(k+1)$ we find that $x_{k+2} - x_{k+1} = -3S$, etc. Now the peak-to-peak amplitude is 5S. However, the fundamental frequency is still $f_S/4$.

Figure 6 shows the steady-state operation of the Song mode of operation. In this mode there is only one possible steady state response since $g_2 = \frac{1}{2}(x_k - x_{k-1})$ when $x_k - x_{k-1} \geq 2S$.

Consider that $x_k - x_{k-1} = 3S$ and $e_{k-1} = +1$ while $e_k = -1$. Then $g_1 = -3S$ while $g_2 = +S$. Note that g_2 should be 1.5S however, due to the use of finite arithmetic we neglect all digits to the right of the binary (or decimal) points. Thus, $x_{k+1} - x_k = -3S + S = -2S$. Since $e_{k+1} = +1$, $g_1 = +2S$ and $g_2 = -S$ and $x_{k+2} - x_{k+1} = +S$; etc.

Note that the peak-to-peak error is 3S and the fundamental frequency of oscillation is $f_s/4$. It is left for the reader to show that this is the only possible steady-state oscill ation.

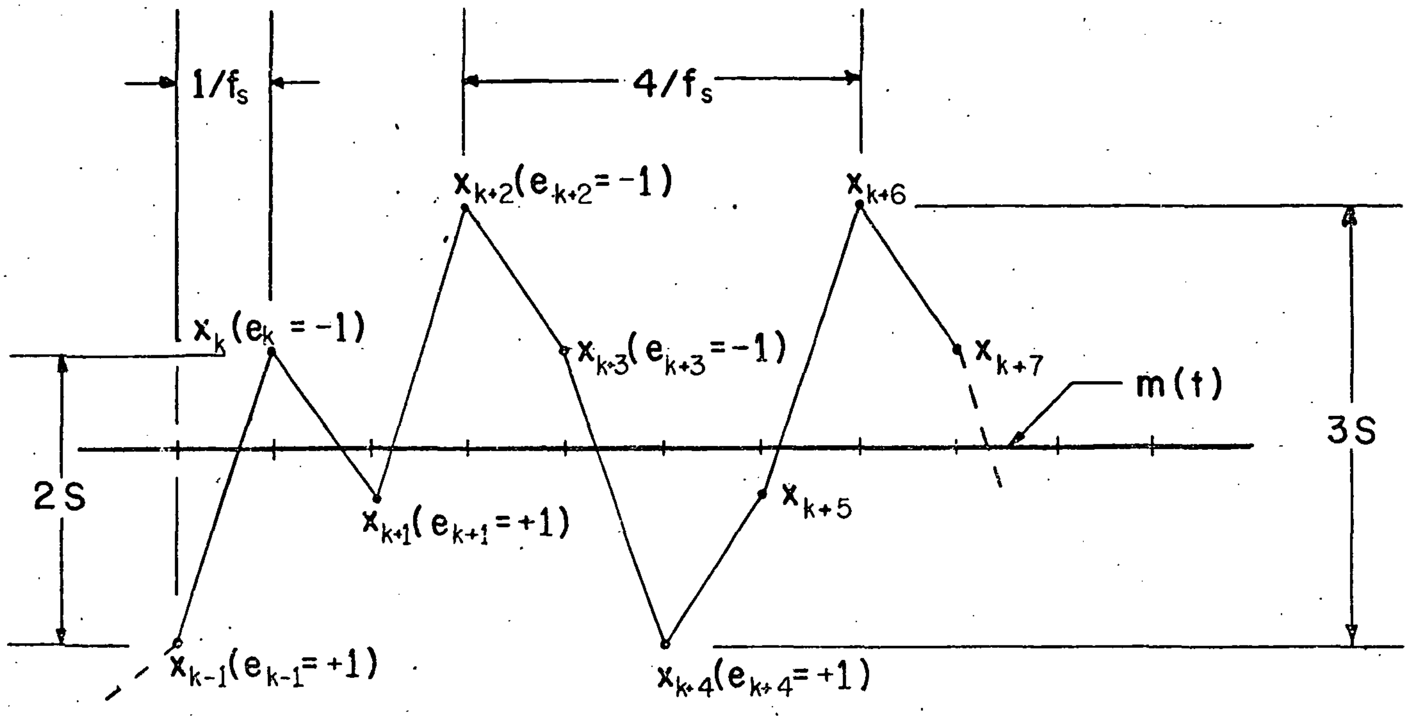
Garodnick's Modification

Garodnick proposed that g_2 be altered to form a new mode of operation defined as

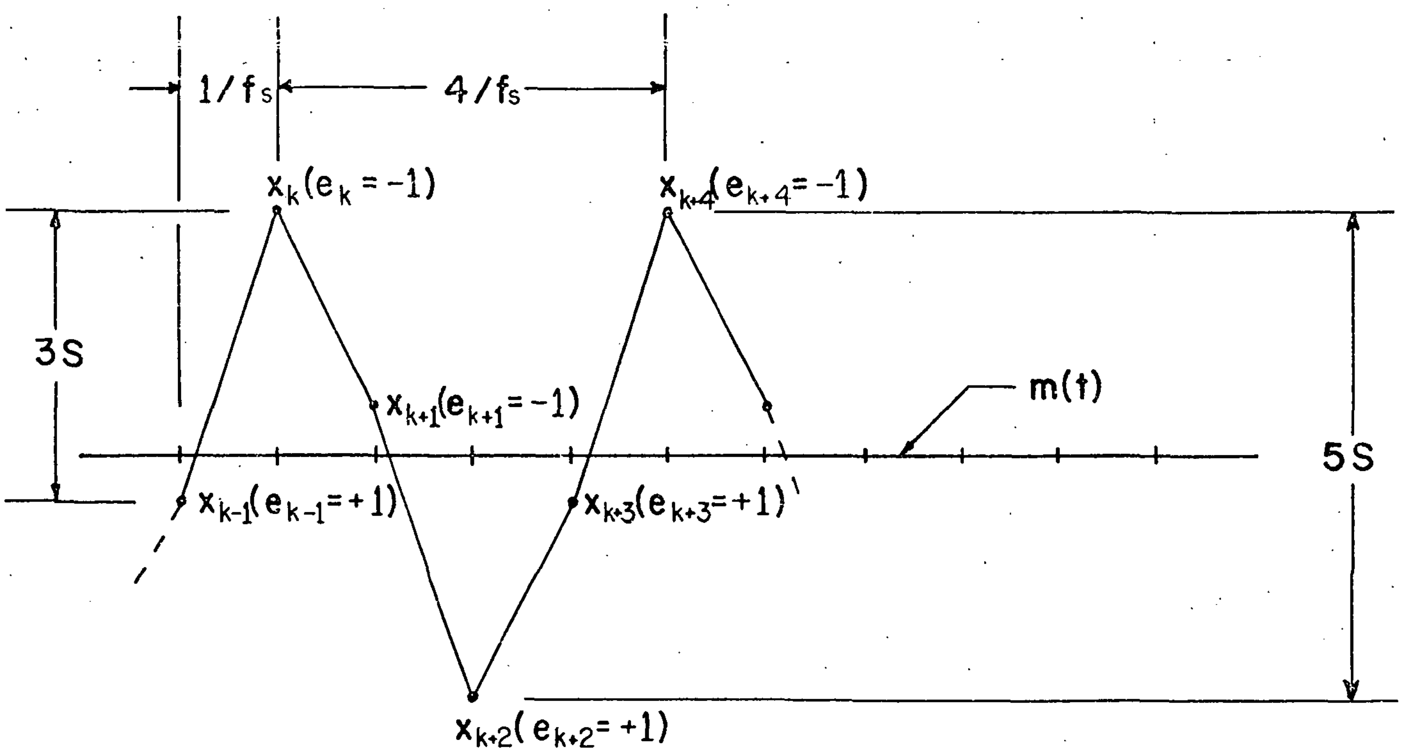
$$g_2 = \text{sgn}\,(e_{k-1} = e_{k-3} + e_{k-3} = \bar{e}_{k-2} + \bar{e}_{k-2} = e_{k-1}) \tag{13}$$

independent of $x_k - x_{k-1}$. Note that this mode of operation requires that three past errors e_{k-1}, e_{k-2} and e_{k-3} be known.

Figure 7 illustrates the transient to steady-state response of the Garodnick Modification of the Abate and Song Modes. The results are compared to the Song mode of operation. Note that in the Song mode the step-size increases and decreases more rapidly. However, the steady-state



(a) $x_k - x_{k-1} = 2S$



(b) $x_k - x_{k-1} = 3S$

Figure 5

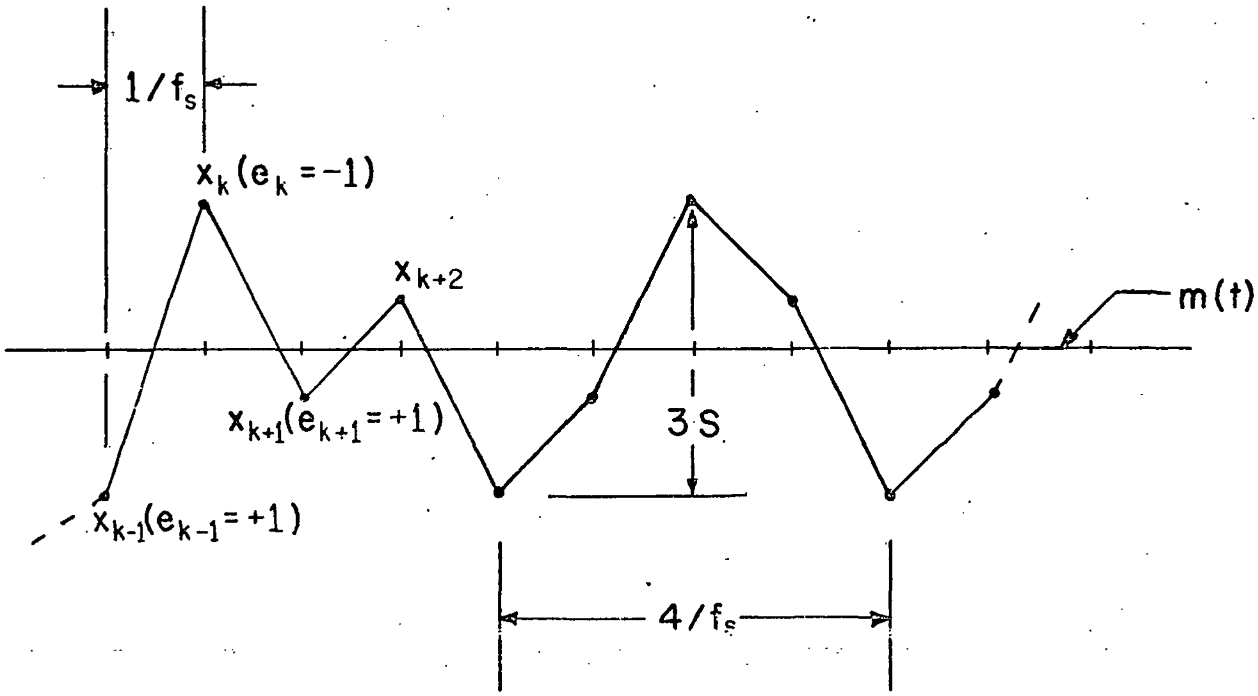


Figure 6 The Song Mode of Operation

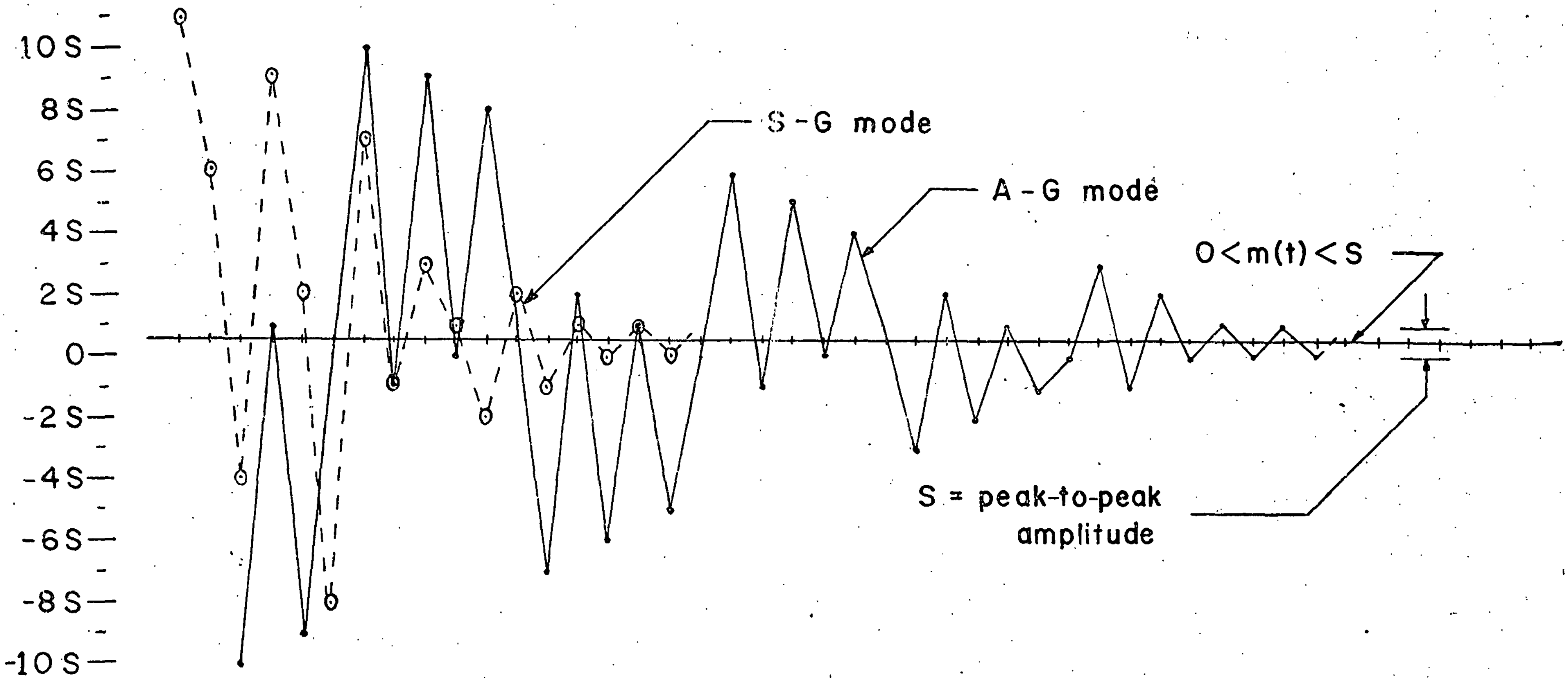
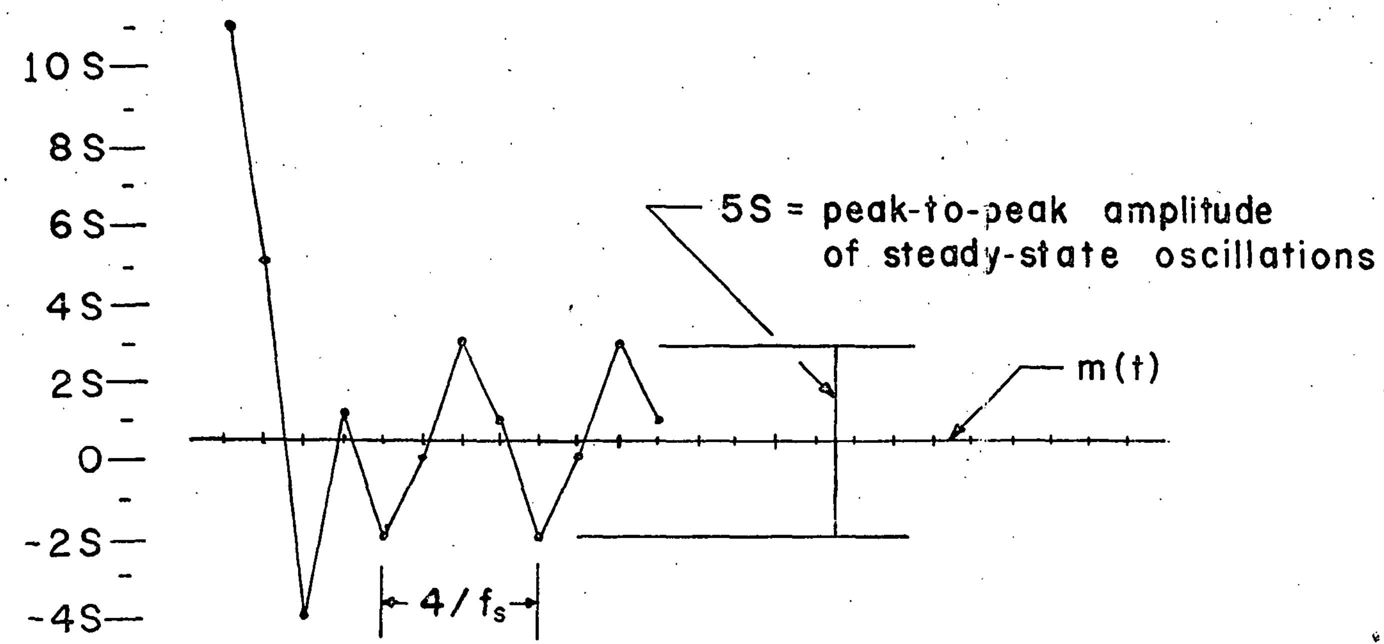

10 S
8 S
6 S
4 S
2 S
0
-2 S
-4 S
-6 S
-8 S
-10 S
S-G mode
A-G mode
0<m(t)<S
S = peak-to-peak
amplitude

Figure 7


10 S
8 S
6 S
4 S
2 S
0
-2 S
-4 S
5S = peak-to-peak amplitude
of steady-state oscillations
m(t)
4 / f_s

response is not desirable as when steady-state is reached with the Garodnick Modification. It should be noted that the example chosen results in oscillations having a peak-to-peak amplitude of 21S if the Abate mode is used. In this example the starting conditions are $e_k = -1$, $e_{k-1} = +1$, $e_{k-2} = +1$, $e_{k-3} = +1$, and $x_{k-1} = 0$ and $x_k = 11S$. The signal $m(t) = +\frac{S}{2}$.

The steps taken in the derivation of the Garodnick Modification of the Abate mode (A - G) are summarized in Table I. The steps in the Garodnick Modification of the Song mode (S - G) are summarized in Table II. The steps of the Song Mode are given in Table III.

Although the Garodnick modification results in an extremely low idle channel noise we have not yet proven that there is an improvement in voice encoding. To do this we are currently incorporating this modification in our experimental DM. Voice tapes will be made and tested. The technique will also be tested for video encoding (the S - G mode will be used.

III Video Encoding Using the Song DM

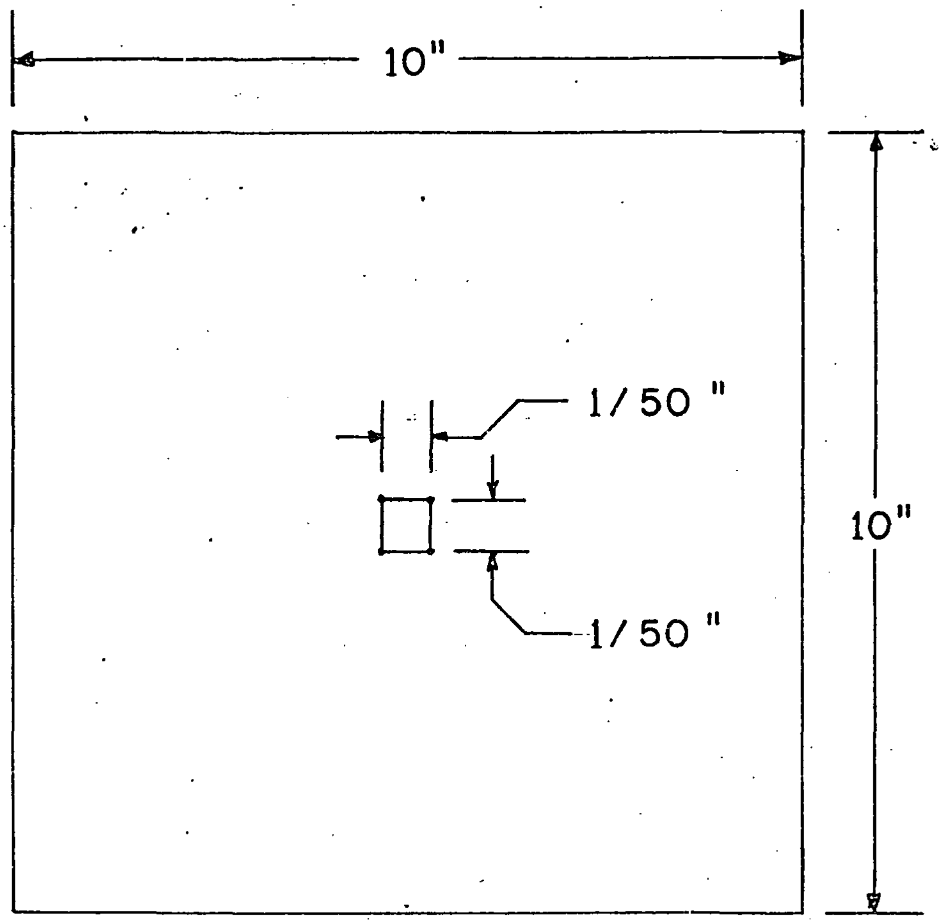
The Song DM can be used to encode video signals since it is a continuous A/D converter with an extremely rapid rate of rise and decay. Experimental verification of this hypothesis has been started.

The experimental system employed is a scaled version of a real system since DTL IC's are used which have a maximum frequency of 1MHz. However, care has been taken to insure the same resolution for both systems.

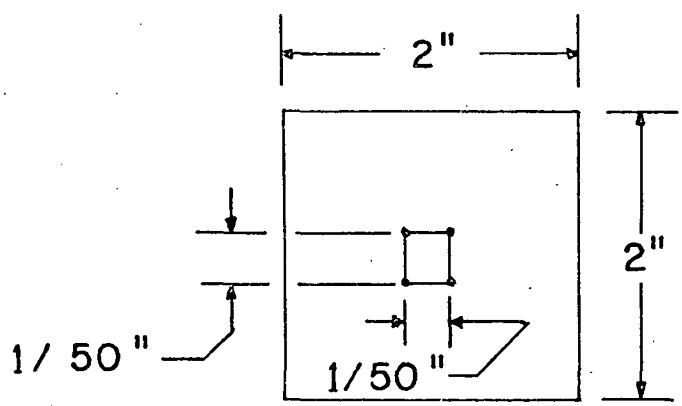
Figure 8a shows a standard TV picture. It is 10 inches square and has a raster of 500 lines. The standard TV picture has a bandwidth of 4 MHz. We see from this figure that for equal horizontal and vertical resolution a PCM system should sample the signal every 1/50 inch at a sampling rate $f_S = 8 \times 10^6$ samples/sec. Thus, the time to trace out one horizontal line is

$$T_H = \frac{1}{8\times10^6}\ \frac{\text{sec}}{\text{sample}} \times \frac{50\ \text{samples}}{\text{inch}} \times \frac{10\ \text{inches}}{\text{line}} = \frac{1}{16\times10^3}\ \frac{\text{sec}}{\text{line}}$$

Thus the line frequency f_M is



(a) Standard Picture: 500 lines - 10" picture



(b) Experimental System: 100 lines - 2" picture

Figure 8

Table I

Steps for A-G Mode $x_k = -10S$

e_{k-3}	e_{k-2}	e_{k-1}	e_k	g_1	g_2	g_1+g_2	x_{k+1}
+	-	-	+	+10S	+S	+11S	+S
-	-	+	-	-11S	+S	-10S	-9S
-	+	-	+	+10S	-S	+9S	0
+	-	+	+	+9S	+S	+10S	10S
-	+	+	-	-10S	-S	-11S	-S
+	+	-	+	+11S	-S	+10S	+9S
+	-	+	-	-10S	+S	-9S	0
-	+	-	+	+9S	-S	+8S	+8S
+	-	+	-	-8S	+S	-7S	+S
-	+	-	-	-7S	-S	-8S	-7S
+	-	-	+	+8S	+S	+9S	+2S
-	-	+	-	-9S	+S	-8S	-6S
-	+	-	+	+8S	-S	+7S	+S
+	-	+	-	-7S	+S	-6S	-5S
-	+	-	+	+6S	-S	+5S	0
+	-	+	+	+5S	+S	+6S	+6S
-	+	+	-	-6S	-S	-7S	-S
+	+	-	+	+7s	-S	+6S	+5S
+	-	+	-	-6S	+S	-5S	0
-	+	-	+	+5S	-S	+4S	4S
+	-	+	-	-4S	+S	-3S	S
-	+	-	-	-3S	-S	-4S	-3S
+	-	-	+	+4S	+S	+5S	+2S
-	-	+	-	-5S	+S	-4S	-2S
-	+	-	+	4S	-S	3S	+S
+	-	+	-	-3S	+S	-2S	-S

Table I (Continued)

e_{k-3}	e_{k-2}	e_{k-1}	e_k	g_1	g_2	g_1+g_2	x_{k+1}
-	+	-	+	+2S	-S	+S	0
+	-	+	+	2S	+S	3S	3S
-	+	+	-	-3S	-S	-4S	-S
+	+	-	+	+4S	-S	3S	+2S
+	-	+	-	-3S	+S	-2S	0
-	+	-	+	2S	-S	S	S
+	-	+	-	-2S	+S	-S	0
-	+	-	+	+2S	-S	+S	S

Table II

Steps for S-G Mode (x_k=11S)

$$g_2 = \frac{1}{2}(x_k - x_{k-1}) \operatorname{sgn} \quad (e_{k-3}\, e_{k-1} + e_{k-1}\, \overline{e}_{k-2} + \overline{e}_{k-2}\, e_{k-3})$$

e_{k-3}	e_{k-2}	e_{k-1}	e_k	g_1	g_2	g_1+g_2	x_{k+1}
+	+	+	-	-11S	+5S	-6S	5S
+	+	-	-	-6S	-3S	-9S	-4S
+	-	-	+	+9S	+4S	+13S	+9S
-	-	+	-	-13S	+6S	-7S	+2S
-	+	-	-	-7S	-3S	-10S	-8S
+	-	-	+	+10S	+5S	15S	+7S
-	-	+	-	-15S	+7S	-8S	-S
-	+	-	+	+8S	-4S	4S	+3S
+	-	+	-	-4S	+2S	-2S	S
-	+	-	-	-2S	-S	-3S	-2S
+	-	-	+	3S	+S	+4S	+2S
-	-	+	-	-4S	+S	-3S	-S
-	+	-	+	+3S	-S	+2S	+S
+	-	+	-	-2S	+S	-S	0
-	+	-	+	2S	-S	+S	+S
+	-	+	-	-2S	+S	-S	0

Table III

Song Mode of Operation ($x_k = 11S$)

e_{k-1}	e_k	g_1	g_2	$g_1 + g_2$	x_{k+1}
+	-	-11S	+5S	-6S	+5S
-	-	-6S	-3S	-9S	-4S
-	+	+9S	-4S	+5S	+S
+	-	-5S	+2S	-3S	-2S
-	+	+3S	-S	+2S	0
+	+	2S	+S	3S	3S
+	-	-3S	+S	-2S	S
-	-	-2S	-S	-3S	-2S
-	+	+3S	-S	+2S	0
+	+	2S	+S	+3S	3S

$$f_M = 16{,}000 \ \frac{\text{lines}}{\text{sec}}$$

as expected. Since 500 lines are traced out for each raster, the raster frequency is

$$f_R = 16{,}000 \ \frac{\text{lines}}{\text{sec}} \times \frac{1\text{raster}}{500 \text{ lines}} = 32 \ \frac{\text{rasters}}{\text{sec}}$$

The scaled system used in our experimental work is shown in Fig 8b. It is 2 inches square and has a raster of 100 lines to maintain the same resolution as the standard TV system. For convenience we select the bandwidth to be 4KHz as this is within our DM capability. Then f_S = 8000 samples/sec. To maintain the resolution of one sample every 1/50 inch we have a line frequency

$$f_H = 8000 \text{ samples/sec.} \times \frac{1 \text{ inch}}{50 \text{ samples}} \times \frac{1 \text{ line}}{2 \text{ inches}}$$

$$= 80 \text{ lines/sec}$$

The raster frequency, f_R is then

$$f_R = 80 \text{ lines/sec} \times \frac{1 \text{ raster}}{100 \text{ lines}} = 0.8 \text{ rasters/sec}$$

The experimental system has been set-up as described above. A 2 inch test pattern provided by NASA is being used with a flying spot scanner. A special purpose low persistance picture tube has been ordered from Tektronix and is due August 1, 1972. As soon as this tube is received, the experimentation will commence.

IV The Double Delta Modulator

The Double Delta Modulator (DDM) significantly increases the signal-to-quantization noise ratio of a DM system. It does this by delta modulating the error signal as shown in Fig 9. It has been shown (1) that when the error is low pass filtered by a sixth-order Butterworth filter and the bit rate of each DM halved, the SNR increases by 16dB. It has also

(1) This research was performed by J. Frank a Doctoral student of mine from P.I.B. Frank's dissertation will be completed in 1972 and the research contained therein published in the final report of this grant (December 1972).

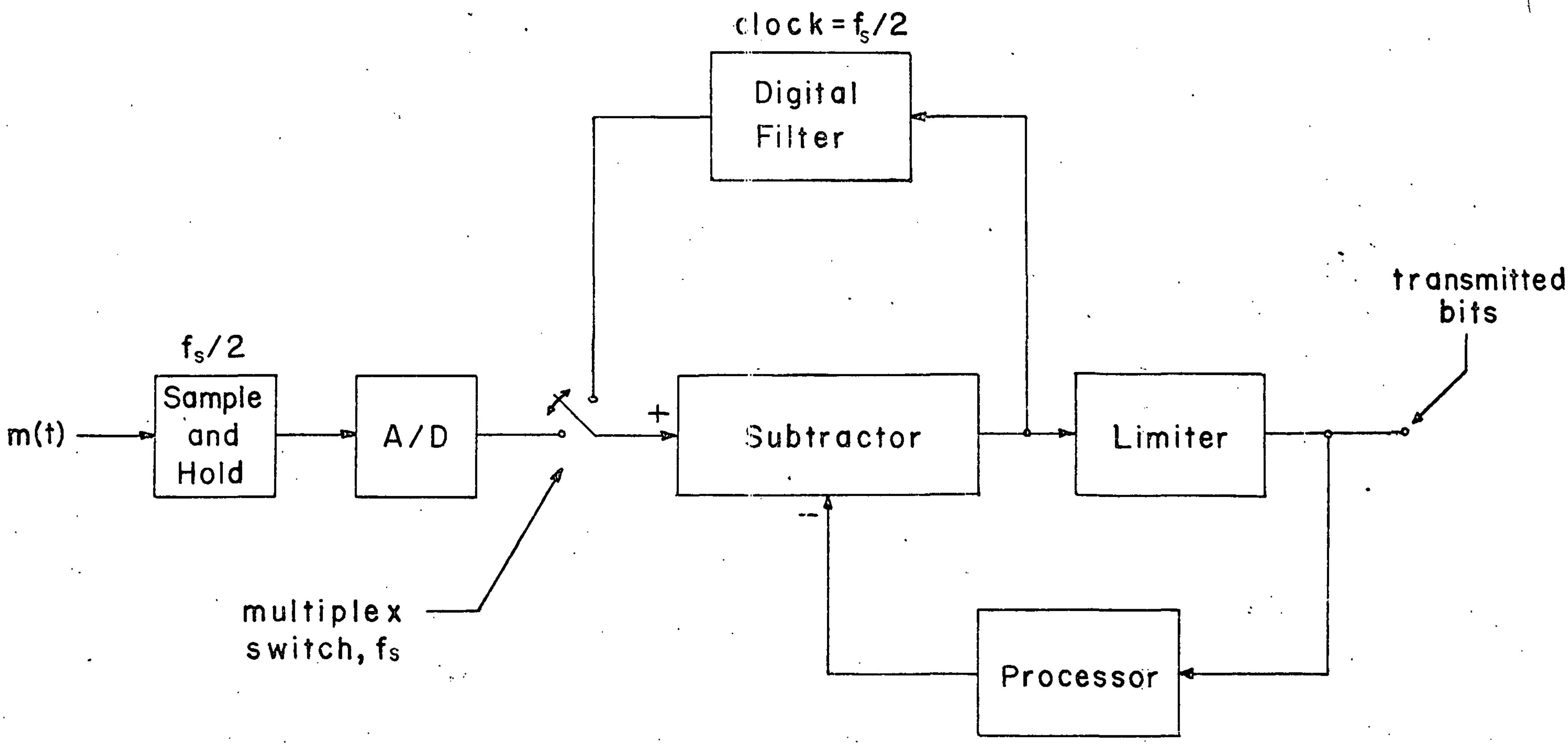


Figure 9 The Double Delta Modulator

been shown that a simple first order filter does not improve performance.

The DDM is currently being constructed digitally. When completed it will operate by time sharing the available ADM. A digital filter is being designed to filter the error signal.

The DDM will be used for voice and video to decrease the transmitted bit rate and hence require a smaller bandwidth.

V.Channel Errors

In order that the DM systems described above be practical, the decoders must be ralatively insensitive to channel errors. We are currently analyzing the error patterns possible with the Song ADM. This is a theoretical analysis which will be followed by experimentation to verify the results obtained.